# Vectorial Quasi-flat Zones for Color Image Simplification

Erhan Aptoula, Jonathan Weber, Sébastien Lefèvre

ISMM 2013
11th International Symposium on Mathematical Morphology
Uppsala, Sweden

May 29th, 2013

# Notations

## Path

A path $\pi(p \rightsquigarrow q)$ of length $N_\pi$ between any two elements $p, q \in E$ is a chain (noted as $\langle \ldots \rangle$) of pairwise adjacent pixels:

$$\pi(p \rightsquigarrow q) \equiv \langle p = p_1, p_2, \ldots, p_{N_\pi - 1}, p_{N_\pi} = q \rangle$$

## Dissimilarity metric

Dissimilarity measured between two pixels $p$ to $q$ is the lowest cost of a path from $p$ to $q$, with the cost of a path being defined as the maximal dissimilarity between pairwise adjacent pixels along the path:

$$\widehat{d}(p,q) = \bigwedge_{\pi \in \Pi} \left\{ \bigvee_{i \in [1,\ldots,N_\pi - 1]} \{ d(p_i, p_{i+1}) \mid \langle p_i, p_{i+1} \rangle \text{subchain of} \pi(p \rightsquigarrow q) \} \right\}$$

with $\Pi$ the set of all possible path between $p$ and $q$

# Notations

## Path

A path $\pi(p \rightsquigarrow q)$ of length $N_\pi$ between any two elements $p, q \in E$ is a chain (noted as $\langle \ldots \rangle$) of pairwise adjacent pixels:

$$\pi(p \rightsquigarrow q) \equiv \langle p = p_1, p_2, \ldots, p_{N_\pi - 1}, p_{N_\pi} = q \rangle$$

## Dissimilarity metric

Dissimilarity measured between two pixels $p$ to $q$ is the lowest cost of a path from $p$ to $q$, with the cost of a path being defined as the maximal dissimilarity between pairwise adjacent pixels along the path:

$$\widehat{d}(p,q) = \bigwedge_{\pi \in \Pi} \left\{ \bigvee_{i \in [1,\ldots,N_\pi - 1]} \{ d(p_i, p_{i+1}) \mid \langle p_i, p_{i+1} \rangle \text{subchain of} \, \pi(p \rightsquigarrow q) \} \right\}$$

with $\Pi$ the set of all possible path between $p$ and $q$

Superpixel approaches are useful operators for image simplification and segmentation (data reduction → CPU reduction).
MM offers several superpixel operators.

Flat Zones are defined as:

$$C(p) = \{p\} \cup \{q | \widehat{d}(p,q) = 0\}$$

149,281 pixels    72,582 flat zones

Flat zones induce heavy oversegmentation

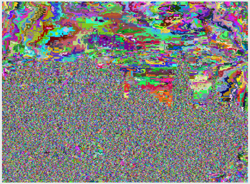
⇒ Unsuitable for efficient image simplification or segmentation

Superpixel approaches are useful operators for image simplification and segmentation (data reduction → CPU reduction).
MM offers several superpixel operators.

Flat Zones are defined as:

$$C(p) = \{p\} \cup \{q | \widehat{d}(p, q) = 0\}$$

149,281 pixels



72,582 flat zones

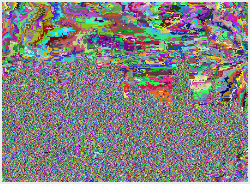
Flat zones induce heavy oversegmentation
⇒ Unsuitable for efficient image simplification or segmentation

Superpixel approaches are useful operators for image simplification and segmentation (data reduction → CPU reduction).
MM offers several superpixel operators.

Flat Zones are defined as:

$$C(p) = \{p\} \cup \{q | \widehat{d}(p,q) = 0\}$$

149,281 pixels



72,582 flat zones

Flat zones induce heavy oversegmentation

⇒ Unsuitable for efficient image simplification or segmentation

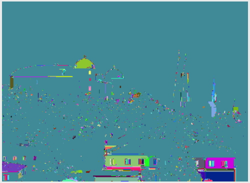
Quasi-Flat Zones $\alpha$ :

- introduction of a local variation criterion ($\alpha$)
  - $\Rightarrow$ produces wider zones

$$C^{\alpha}(p) = \{p\} \cup \{q | \widehat{d}(p,q) \leq \alpha\}$$

149,281 pixels

11,648 QFZ ($\alpha = 5$)



2,813 QFZ ($\alpha = 10$)

Quasi-Flat zones $\alpha$ reduce oversegmentation

$\Rightarrow$ quickly induces undersegmentation (chaining-effect)

Quasi-Flat Zones $\alpha$ :

- introduction of a local variation criterion ($\alpha$)
  - $\Rightarrow$ produces wider zones

$$C^{\alpha}(p) = \{p\} \cup \{q | \widehat{d}(p,q) \leq \alpha\}$$

149,281 pixels

11,648 QFZ ($\alpha = 5$)

2,813 QFZ ($\alpha = 10$)

Quasi-Flat zones $\alpha$ reduce oversegmentation

$\Rightarrow$ quickly induces undersegmentation (chaining-effect)

Quasi-Flat Zones $\alpha$ :

- introduction of a local variation criterion ($\alpha$)
  - $\Rightarrow$ produces wider zones

$$C^{\alpha}(p) = \{p\} \cup \{q | \widehat{d}(p,q) \leq \alpha\}$$

149,281 pixels

11,648 QFZ ($\alpha = 5$)

2,813 QFZ ($\alpha = 10$)

Quasi-Flat zones $\alpha$ reduce oversegmentation

$\Rightarrow$ quickly induces undersegmentation (chaining-effect)

Quasi-flat zone $\alpha,\omega$ :

- introduction of a global variation criterion ($\omega$)
  - $\Rightarrow$ counters the chaining-effect
- Idea : find highest $\alpha$ that satisfies constraint $\omega$

$$C^{\alpha,\omega}(p) = \max\{C^{\alpha'}(p) \mid \alpha' \leq \alpha \text{ and } R(C^{\alpha'}(p)) \leq \omega\}$$

with $R(C^{\alpha})$ the maximal difference between pixels attributes of $C^{\alpha}$

149,281 pixels

16,865 QFZ

$\alpha/\omega = 50$

8,958 QFZ

$\alpha/\omega = 75$

Quasi-Flat Zones $\alpha,\omega$ greatly reduce oversegmentation

$\Rightarrow$ suffers less from undersegmentation

Quasi-flat zone $\alpha,\omega$ :

- introduction of a global variation criterion ($\omega$)
  - $\Rightarrow$ counters the chaining-effect
- Idea : find highest $\alpha$ that satisfies constraint $\omega$

$$C^{\alpha,\omega}(p) = \max\{C^{\alpha'}(p) \mid \alpha' \leq \alpha \text{ and } R(C^{\alpha'}(p)) \leq \omega\}$$

with $R(C^{\alpha})$ the maximal difference between pixels attributes of $C^{\alpha}$

149,281 pixels

16,865 QFZ
$\alpha/\omega = 50$

8,958 QFZ
$\alpha/\omega = 75$

Quasi-Flat Zones $\alpha,\omega$ greatly reduce oversegmentation

$\Rightarrow$ suffers less from undersegmentation

Quasi-flat zone $\alpha,\omega$ :

- introduction of a global variation criterion ($\omega$)
  - $\Rightarrow$ counters the chaining-effect
- Idea : find highest $\alpha$ that satisfies constraint $\omega$

$$C^{\alpha,\omega}(p) = \max\{C^{\alpha'}(p) \mid \alpha' \leq \alpha \text{ and } R(C^{\alpha'}(p)) \leq \omega\}$$

with $R(C^{\alpha})$ the maximal difference between pixels attributes of $C^{\alpha}$

149,281 pixels

16,865 QFZ
$\alpha/\omega = 50$

8,958 QFZ
$\alpha/\omega = 75$

Quasi-Flat Zones $\alpha,\omega$ greatly reduce oversegmentation

$\Rightarrow$ suffers less from undersegmentation

# What about QFZ in color images ?

QFZ are well-defined for grayscale images as gray images are composed of ordered scalar values. In fact, QFZ needs :

- ordered values (search of the highest $\alpha$)
- existence of a difference operator (computation of $\widehat{d}(p, q)$)

In color images, we are dealing with vector values that are no longer naturally ordered

$\Rightarrow$ QFZ extension to color images is not straightforward

# What about QFZ in color images ?

QFZ are well-defined for grayscale images as gray images are composed of ordered scalar values. In fact, QFZ needs :

- ordered values (search of the highest $\alpha$)
- existence of a difference operator (computation of $\widehat{d}(p,q)$)

In color images, we are dealing with vector values that are no longer naturally ordered

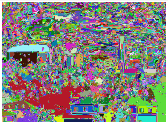
$\Rightarrow$ QFZ extension to color images is not straightforward

# Quasi-Marginal Approach [Weber, PhD Th., 2011]

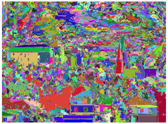
**Idea**:

1. Process QFZ marginally
2. Merge them using a voting mechanism (parameter $\iota$)

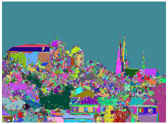
original
149,281 pix.



$C^{\alpha,\omega}_{red}$
20,198 QFZ



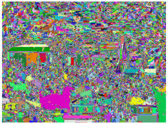
$C^{\alpha,\omega}_{green}$
18,443 QFZ



$C^{\alpha,\omega}_{blue}$
12,400 QFZ

$C^{\alpha,\omega}_{\iota-Marginal}$ $(\iota = 1)$
3,141 QFZ

$C^{\alpha,\omega}_{\iota-Marginal}$ $(\iota = 2)$
13,131 QFZ



$C^{\alpha,\omega}_{\iota-Marginal}$ $(\iota = 3)$
38,435 QFZ

⇒ Low $\iota$ values lead to undersegmentation

⇒ High $\iota$ values lead to oversegmentation

# Quasi-Marginal Approach [Weber, PhD Th., 2011]

**Idea**:

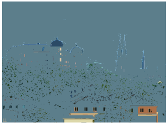
1. Process QFZ marginally
2. Merge them using a voting mechanism (parameter $\iota$)

original
149,281 pix.

$C^{\alpha,\omega}_{red}$
20,198 QFZ

$C^{\alpha,\omega}_{green}$
18,443 QFZ

$C^{\alpha,\omega}_{blue}$
12,400 QFZ



$C^{\alpha,\omega}_{\iota-Marginal}$ $(\iota = 1)$
3,141 QFZ

$C^{\alpha,\omega}_{\iota-Marginal}$ $(\iota = 2)$
13,131 QFZ

$C^{\alpha,\omega}_{\iota-Marginal}$ $(\iota = 3)$
38,435 QFZ

$\Rightarrow$ Low $\iota$ values lead to undersegmentation

$\Rightarrow$ High $\iota$ values lead to oversegmentation

# Quasi-Marginal Approach [Weber, PhD Th., 2011]

**Idea**:

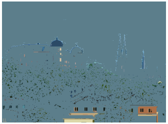
1. Process QFZ marginally
2. Merge them using a voting mechanism (parameter $\iota$)

original
149,281 pix.

$C^{\alpha,\omega}_{red}$
20,198 QFZ

$C^{\alpha,\omega}_{green}$
18,443 QFZ

$C^{\alpha,\omega}_{blue}$
12,400 QFZ



$C^{\alpha,\omega}_{\iota-Marginal}$ $(\iota = 1)$
3,141 QFZ

$C^{\alpha,\omega}_{\iota-Marginal}$ $(\iota = 2)$
13,131 QFZ

$C^{\alpha,\omega}_{\iota-Marginal}$ $(\iota = 3)$
38,435 QFZ

$\Rightarrow$ Low $\iota$ values lead to undersegmentation

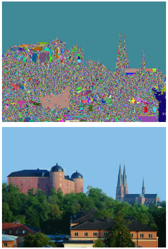
$\Rightarrow$ High $\iota$ values lead to oversegmentation

# Customized metrics approach [Zanoguera, PhD Th., 2001]

**Idea**: Introduce a scalar metric to compare pixel values

⇒ e.g. L2 for RGB images

original
149,281 pix.



$C^{\alpha}_{L2}$ ($\alpha = 4$)
50,234 QFZ

$C^{\alpha}_{L2}$ ($\alpha = 8$)
23,435 QFZ

$C^{\alpha}_{L2}$ ($\alpha = 12$)
10,708 QFZ

but has to face the chaining-effect (because limited to $C^{\alpha}$)

⇒ Hardly applicable to global range criterion $\omega$

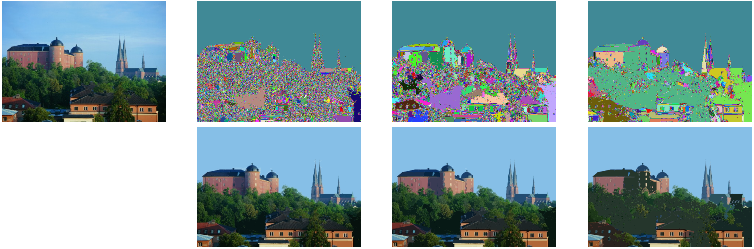
⇒ Vector values are not ordered, so no min/max values

⇒ $\omega$ computation is in quadratic complexity

# Customized metrics approach [Zanoguera, PhD Th., 2001]

**Idea**: Introduce a scalar metric to compare pixel values

$\Rightarrow$ e.g. L2 for RGB images



original 149,281 pix. | $C^{\alpha}_{L2}$ ($\alpha = 4$) 50,234 QFZ | $C^{\alpha}_{L2}$ ($\alpha = 8$) 23,435 QFZ | $C^{\alpha}_{L2}$ ($\alpha = 12$) 10,708 QFZ

but has to face the chaining-effect (because limited to $C^{\alpha}$)

$\Rightarrow$ Hardly applicable to global range criterion $\omega$

$\Rightarrow$ Vector values are not ordered, so no min/max values

$\Rightarrow$ $\omega$ computation is in quadratic complexity

# Semi-vectorial approach [Soille, PAMI, 2008]

**Idea**: Introduce vectorial parameters, each criterion has to be satisfied independently for each channel:

$$\forall i \in [1,n], \forall p,q \in E, |f^i(p) - f^i(q)| \leq \alpha_i \Leftrightarrow d(p,q) \leq \boldsymbol{\alpha}$$

**Question**: How to determine highest $\boldsymbol{\alpha}$ that satisfies global range criterion $(\omega)$?

- ⚠ How to order [2,1,1], [1,2,1] and [1,1,2]?
- ⇒ By considering only $\alpha$ of type $[x,x,x]$
- ⇒ Inducing a total ordering in this subspace, $[0,0,0] < [1,1,1] < [2,2,2] < \ldots$

# Semi-vectorial approach [Soille, PAMI, 2008]

**Idea**: Introduce vectorial parameters, each criterion has to be satisfied independently for each channel:

$$\forall i \in [1,n], \forall p,q \in E, |f^i(p) - f^i(q)| \leq \alpha_i \Leftrightarrow d(p,q) \leq \boldsymbol{\alpha}$$

**Question**: How to determine highest $\boldsymbol{\alpha}$ that satisfies global range criterion ($\boldsymbol{\omega}$)?

- ⚠ How to order [2,1,1], [1,2,1] and [1,1,2]?
- ⇒ By considering only $\boldsymbol{\alpha}$ of type $[x,x,x]$
- ⇒ Inducing a total ordering in this subspace, $[0,0,0] < [1,1,1] < [2,2,2] < \dots$

# Semi-vectorial approach [Soille, PAMI, 2008]

**Idea**: Introduce vectorial parameters, each criterion has to be satisfied independently for each channel:

$$\forall i \in [1,n], \forall p,q \in E, |f^i(p) - f^i(q)| \leq \alpha_i \Leftrightarrow d(p,q) \leq \boldsymbol{\alpha}$$

**Question**: How to determine highest $\boldsymbol{\alpha}$ that satisfies global range criterion $(\boldsymbol{\omega})$?

⚠ How to order [2,1,1], [1,2,1] and [1,1,2]?

⇒ By considering only $\boldsymbol{\alpha}$ of type $[x,x,x]$

⇒ Inducing a total ordering in this subspace, $[0,0,0] < [1,1,1] < [2,2,2] < \ldots$

# Semi-vectorial approach [Soille, PAMI, 2008]

**Idea**: Introduce vectorial parameters, each criterion has to be satisfied independently for each channel:

$$\forall i \in [1,n], \forall p,q \in E, |f^i(p) - f^i(q)| \leq \alpha_i \Leftrightarrow d(p,q) \leq \boldsymbol{\alpha}$$

**Question**: How to determine highest $\boldsymbol{\alpha}$ that satisfies global range criterion ($\boldsymbol{\omega}$)?

- ⚠ How to order [2,1,1], [1,2,1] and [1,1,2]?
- ⇒ By considering only $\boldsymbol{\alpha}$ of type $[x,x,x]$
- ⇒ Inducing a total ordering in this subspace, $[0,0,0] < [1,1,1] < [2,2,2] < \ldots$

# Semi-vectorial approach [Soille, PAMI, 2008]

**Idea**: Introduce vectorial parameters, each criterion has to be satisfied independently for each channel:

$$\forall i \in [1,n],\ \forall p,q \in E,\ |f^i(p) - f^i(q)| \leq \alpha_i \Leftrightarrow d(p,q) \leq \boldsymbol{\alpha}$$

**Question**: How to determine highest $\boldsymbol{\alpha}$ that satisfies global range criterion $(\boldsymbol{\omega})$?

- ⚠ How to order $[2,1,1]$, $[1,2,1]$ and $[1,1,2]$?
- ⇒ By considering only $\boldsymbol{\alpha}$ of type $[x,x,x]$
- ⇒ Inducing a total ordering in this subspace, $[0,0,0] < [1,1,1] < [2,2,2] < \ldots$

# Semi-vectorial approach [Soille, PAMI, 2008]

original 149,281 pix. | $C^{\alpha,\omega}_{Soille}$ $(\alpha/\omega = 40)$ 25,803 QFZ | $C^{\alpha,\omega}_{Soille}$ $(\alpha/\omega = 80)$ 13,339 QFZ | $C^{\alpha,\omega}_{Soille}$ $(\alpha/\omega = 120)$ 7,215 QFZ

Applicable to a global range criterion but small search space for highest $\alpha$

- ⚠ Only 256 possible $\alpha$ values instead of 16,777,216 colors in RGB images
- ⇒ As higher $\alpha$ means wider QFZ, this sub-quantization may hidden the best $\alpha$ values for oversegmentation reduction

# Semi-vectorial approach [Soille, PAMI, 2008]

| original | $C^{\alpha,\omega}_{Soille}$ $(\alpha/\omega = 40)$ | $C^{\alpha,\omega}_{Soille}$ $(\alpha/\omega = 80)$ | $C^{\alpha,\omega}_{Soille}$ $(\alpha/\omega = 120)$ |
|---|---|---|---|
| 149,281 pix. | 25,803 QFZ | 13,339 QFZ | 7,215 QFZ |

Applicable to a global range criterion but small search space for highest $\boldsymbol{\alpha}$

⚠ Only 256 possible $\boldsymbol{\alpha}$ values instead of 16,777,216 colors in RGB images

⇒ As higher $\boldsymbol{\alpha}$ means wider QFZ, this sub-quantization may hidden the best $\alpha$ values for oversegmentation reduction

# Motivation

Using different $\alpha/\omega$ values per channel:

- provides a higher level of customization
  - $\Rightarrow$ tuned for application context
- provides a finer search space for $C^{\alpha,\omega}$
  - $\Rightarrow$ wider QFZ (due to higher $\boldsymbol{\alpha}$ for a given $\boldsymbol{\omega}$)

How to deal with vectorial $\boldsymbol{\alpha}$ and $\boldsymbol{\omega}$?

$\Rightarrow$ Modify pixel attribute difference $\widehat{d}$ into $\widehat{d}_{\leq}$ comparable with $\boldsymbol{\alpha}$

$\Rightarrow$ Solution should be adapted to any arbitrary ordering ($\leq$)

# Motivation

Using different $\alpha/\omega$ values per channel:

- provides a higher level of customization
  - $\Rightarrow$ tuned for application context
- provides a finer search space for $C^{\alpha,\omega}$
  - $\Rightarrow$ wider QFZ (due to higher $\boldsymbol{\alpha}$ for a given $\boldsymbol{\omega}$)

How to deal with vectorial $\boldsymbol{\alpha}$ and $\boldsymbol{\omega}$?

$\Rightarrow$ Modify pixel attribute difference $\widehat{d}$ into $\widehat{d}_{\preceq}$ comparable with $\boldsymbol{\alpha}$

$\Rightarrow$ Solution should be adapted to any arbitrary ordering ($\preceq$)

How to implement such $\widehat{d}_{\preceq}$?

- $\Rightarrow$ Rely on a rank operator such as $\text{rank}_{\preceq} : T \to \mathbb{N}$
  which associates each vector with its position in the space $T$ w.r.t. $\preceq$
- $\Rightarrow$ This rank operator needs also to be applied to $\boldsymbol{\alpha}$

$$\forall p, q \in E,\ d_{\preceq}(p,q) = \left|\text{rank}_{\preceq}(\boldsymbol{f}(p)) - \text{rank}_{\preceq}(\boldsymbol{f}(q))\right|$$

**Vectorial $C^{\alpha}$**

$$C^{\boldsymbol{\alpha}}_{\preceq}(p) = \{p\} \cup \{q \mid \widehat{\mathbf{d}_{\preceq}}(p,q) \leq \text{rank}_{\preceq}(\boldsymbol{\alpha})\}$$

**Implementation** :

1. Transform color image in rank image (using precomputed look up table).
2. Apply greylevel $C^{\alpha}$ on the rank image with $\alpha = rank_{\preceq}(\boldsymbol{\alpha})$

How to implement such $\widehat{d}_{\preceq}$?

⇒ Rely on a rank operator such as $\text{rank}_{\preceq} : T \rightarrow \mathbb{N}$
which associates each vector with its position in the space $T$ w.r.t. $\preceq$

⇒ This rank operator needs also to be applied to $\boldsymbol{\alpha}$

$$\forall p, q \in E,\ d_{\preceq}(p,q) = \left|\text{rank}_{\preceq}(\boldsymbol{f}(p)) - \text{rank}_{\preceq}(\boldsymbol{f}(q))\right|$$

## Vectorial $C^{\alpha}$

$$C_{\preceq}^{\boldsymbol{\alpha}}(p) = \{p\} \cup \{q \mid \widehat{\mathrm{d}_{\preceq}}(p,q) \leq \text{rank}_{\preceq}(\boldsymbol{\alpha})\}$$

**Implementation** :

1. Transform color image in rank image (using precomputed look up table).
2. Apply greylevel $C^{\alpha}$ on the rank image with $\alpha = rank_{\preceq}(\boldsymbol{\alpha})$

How to implement such $\widehat{d}_{\preceq}$?

⇒ Rely on a rank operator such as $\text{rank}_{\preceq} : T \to \mathbb{N}$
which associates each vector with its position in the space $T$ w.r.t. $\preceq$

⇒ This rank operator needs also to be applied to $\boldsymbol{\alpha}$

$$\forall p, q \in E,\ d_{\preceq}(p,q) = \left|\text{rank}_{\preceq}(\boldsymbol{f}(p)) - \text{rank}_{\preceq}(\boldsymbol{f}(q))\right|$$

**Vectorial $C^{\alpha}$**

$$C^{\boldsymbol{\alpha}}_{\preceq}(p) = \{p\} \cup \{q \mid \widehat{\mathbf{d}_{\preceq}}(p,q) \leq \text{rank}_{\preceq}(\boldsymbol{\alpha})\}$$

**Implementation** :

1. Transform color image in rank image (using precomputed look up table).
2. Apply greylevel $C^{\alpha}$ on the rank image with $\alpha = rank_{\preceq}(\boldsymbol{\alpha})$

How to implement such $\widehat{d}_{\preceq}$?

⇒ Rely on a rank operator such as $\text{rank}_{\preceq} : T \to \mathbb{N}$
which associates each vector with its position in the space $T$ w.r.t. $\preceq$

⇒ This rank operator needs also to be applied to $\boldsymbol{\alpha}$

$$\forall p, q \in E,\ d_{\preceq}(p,q) = \left|\text{rank}_{\preceq}(\boldsymbol{f}(p)) - \text{rank}_{\preceq}(\boldsymbol{f}(q))\right|$$

## Vectorial $C^{\alpha}$

$$C_{\preceq}^{\boldsymbol{\alpha}}(p) = \{p\} \cup \{q \mid \widehat{\mathbf{d}_{\preceq}}(p,q) \leq \text{rank}_{\preceq}(\boldsymbol{\alpha})\}$$

**Implementation** :

1. Transform color image in rank image (using precomputed look up table).
2. Apply greylevel $C^{\alpha}$ on the rank image with $\alpha = rank_{\preceq}(\boldsymbol{\alpha})$

## Purely vectorial $C^{\alpha,\omega}$

$$C_{\leq}^{\boldsymbol{\alpha},\boldsymbol{\omega}}(p) = \max\{C_{\leq}^{\boldsymbol{\alpha}'}(p) \mid \boldsymbol{\alpha}' \preceq \boldsymbol{\alpha} \text{ and } R_{\leq}(C^{\boldsymbol{\alpha}'}(p)) \leq \mathrm{rank}_{\leq}(\boldsymbol{\omega})\}$$

⚠ Total vector ordering is needed for $\{\boldsymbol{\alpha}\}$ to find $\boldsymbol{\alpha}'$!

- Enables the computation of color QFZ using vector parameters
- Enables customization (inter-channel relation modeling, channel-specific parameters, . . . )
- Preserves theoretical properties of QFZ
- Compatible with existing greylevel implementations

⚠ Parameter settings is not trivial

  - Choice of vectorial ordering
  - Setting of $\boldsymbol{\alpha}$ and $\boldsymbol{\omega}$

## Purely vectorial $C^{\alpha,\omega}$

$$C_{\preceq}^{\boldsymbol{\alpha},\boldsymbol{\omega}}(p) = \max\{C_{\preceq}^{\boldsymbol{\alpha}'}(p) \mid \boldsymbol{\alpha}' \preceq \boldsymbol{\alpha} \text{ and } R_{\preceq}(C^{\boldsymbol{\alpha}'}(p)) \leq \text{rank}_{\preceq}(\boldsymbol{\omega})\}$$

⚠ Total vector ordering is needed for $\{\boldsymbol{\alpha}\}$ to find $\boldsymbol{\alpha}'$!

- Enables the computation of color QFZ using vector parameters
- Enables customization (inter-channel relation modeling, channel-specific parameters, . . . )
- Preserves theoretical properties of QFZ
- Compatible with existing greylevel implementations

⚠ Parameter settings is not trivial

- Choice of vectorial ordering
- Setting of $\boldsymbol{\alpha}$ and $\boldsymbol{\omega}$

# Experimental Setup

**Data** : Berkeley Segmentation Dataset
**Ordering** : RGB $L_2$ norm
(+ lexicographical comparison $\leq_L$ to avoid preordering)

$$\forall\, \mathbf{v}, \mathbf{v}' \in \mathbb{R}^3,\ \ \mathbf{v} \preceq_{rgb} \mathbf{v}' \Leftrightarrow [\|\mathbf{v}\|, v_1, v_2, v_3]^T \leq_L [\|\mathbf{v}'\|, v'_1, v'_2, v'_3]^T$$

**Quality metric (segmentation task)** :

- a set of reference regions is known
- *Maximal precision* (MP)
  ratio of well segmented pixels (following assignment of each QFZ to the most overlapping reference region)
- *Oversegmentation ratio* (OSR)
  ratio between # QFZ and # reference regions

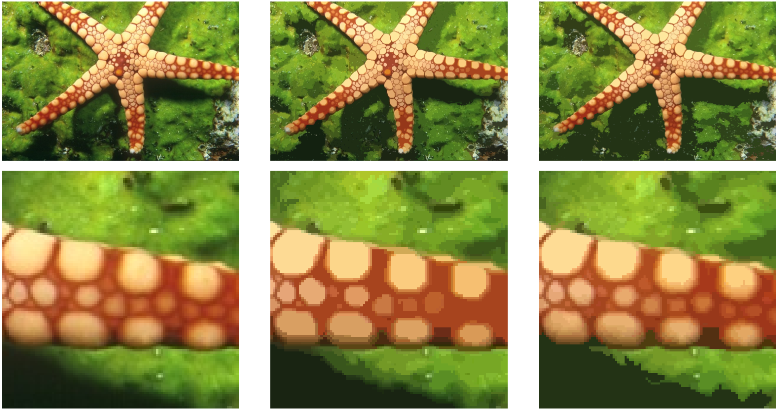
# Image Simplification on Berkeley Segmentation Dataset

Original
154,401 pixels

$C^{\alpha,\omega}_{Soille}$
24,461 QFZ

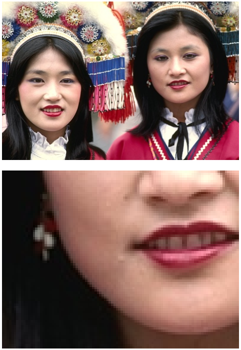
$C^{\boldsymbol{\alpha},\boldsymbol{\omega}}_{\preceq_{RGB}}$
25,846 QFZ

# Image Simplification on Berkeley Segmentation Dataset

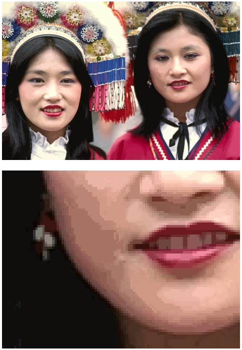


| Original | $C^{\alpha,\omega}_{Soille}$ | $C^{\boldsymbol{\alpha},\boldsymbol{\omega}}_{\preceq_{RGB}}$ |
|---|---|---|
| 154,401 pixels | 22,592 QFZ | 21,576 QFZ |

# Image Simplification on Berkeley Segmentation Dataset



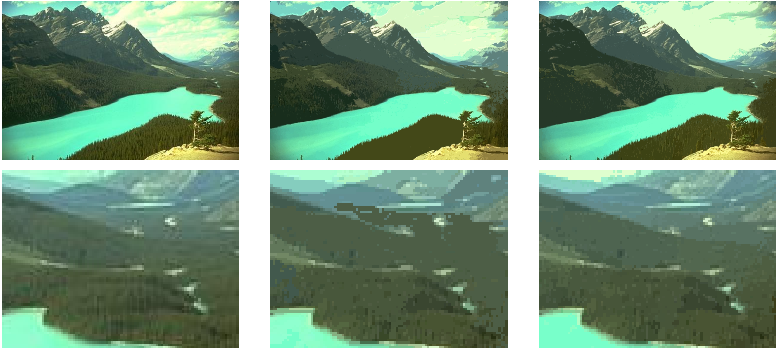
Original
154,401 pixels



$C^{\alpha,\omega}_{Soille}$
33,182 QFZ

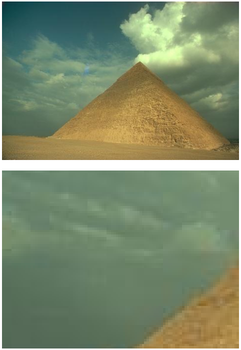
$C^{\boldsymbol{\alpha},\boldsymbol{\omega}}_{\preceq_{RGB}}$
31,155 QFZ

# Image Simplification on Berkeley Segmentation Dataset

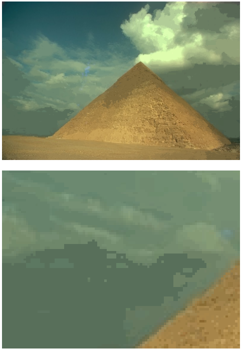


| Original | $C^{\alpha,\omega}_{Soille}$ | $C^{\boldsymbol{\alpha},\boldsymbol{\omega}}_{\preceq_{RGB}}$ |
|---|---|---|
| 154,401 pixels | 20,931 QFZ | 20,496 QFZ |

# Image Simplification on Berkeley Segmentation Dataset



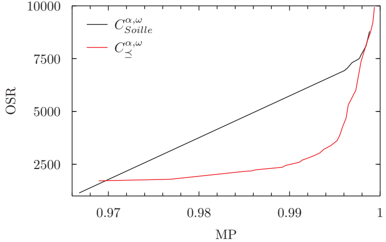
Original
154,401 pixels



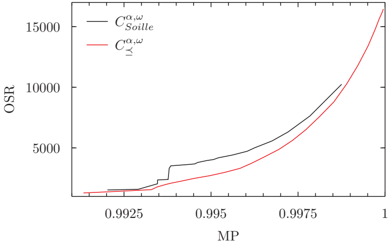
$C^{\alpha,\omega}_{Soille}$
29,766 QFZ

$C^{\boldsymbol{\alpha},\boldsymbol{\omega}}_{\preceq_{RGB}}$
28,562 QFZ

# Segmentation on Berkeley Segmentation Dataset







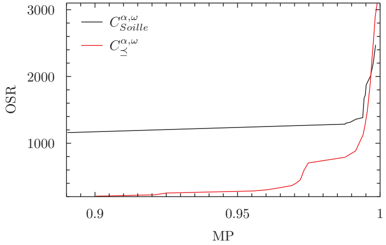


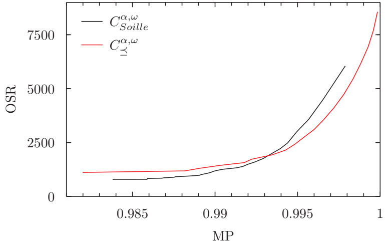
Original

MP vs. OSR

# Segmentation on Berkeley Segmentation Dataset

Original




MP vs. OSR

1. Context
2. State-of-the-art
3. Vectorial Quasi-Flat Zones
4. Experiments
5. Conclusion and Perspectives

## Conclusion

- Our approach achieves good results:
  - Image simplification
  - Image segmentation
- Preserves theoretical properties of QFZ
- Straightforward implementation from grey-level version
- But choice of vector ordering / parameters is not intuitive

## Perspectives

- Automatically determine:
  - Optimal color ordering
  - Vector parameters
- Achieve wider and more rigorous experimentation
- Embed this development into framework of morphological trees

## Conclusion

- Our approach achieves good results:
  - Image simplification
  - Image segmentation
- Preserves theoretical properties of QFZ
- Straightforward implementation from grey-level version
- But choice of vector ordering / parameters is not intuitive

## Perspectives

- Automatically determine:
  - Optimal color ordering
  - Vector parameters
- Achieve wider and more rigorous experimentation
- Embed this development into framework of morphological trees

## Conclusion

- Our approach achieves good results:
  - Image simplification
  - Image segmentation
- Preserves theoretical properties of QFZ
- Straightforward implementation from grey-level version
- But choice of vector ordering / parameters is not intuitive

## Perspectives

- Automatically determine:
  - Optimal color ordering
  - Vector parameters
- Achieve wider and more rigorous experimentation
- Embed this development into framework of morphological trees

## Conclusion

- Our approach achieves good results:
  - Image simplification
  - Image segmentation
- Preserves theoretical properties of QFZ
- Straightforward implementation from grey-level version
- But choice of vector ordering / parameters is not intuitive

## Perspectives

- Automatically determine:
  - Optimal color ordering
  - Vector parameters
- Achieve wider and more rigorous experimentation
- Embed this development into framework of morphological trees

## Conclusion

- Our approach achieves good results:
  - Image simplification
  - Image segmentation
- Preserves theoretical properties of QFZ
- Straightforward implementation from grey-level version
- But choice of vector ordering / parameters is not intuitive

## Perspectives

- Automatically determine:
  - Optimal color ordering
  - Vector parameters
- Achieve wider and more rigorous experimentation
- Embed this development into framework of morphological trees

## Conclusion

- Our approach achieves good results:
  - Image simplification
  - Image segmentation
- Preserves theoretical properties of QFZ
- Straightforward implementation from grey-level version
- But choice of vector ordering / parameters is not intuitive

## Perspectives

- Automatically determine:
  - Optimal color ordering
  - Vector parameters
- Achieve wider and more rigorous experimentation
- Embed this development into framework of morphological trees

## Conclusion

- Our approach achieves good results:
  - Image simplification
  - Image segmentation
- Preserves theoretical properties of QFZ
- Straightforward implementation from grey-level version
- But choice of vector ordering / parameters is not intuitive

## Perspectives

- Automatically determine:
  - Optimal color ordering
  - Vector parameters
- Achieve wider and more rigorous experimentation
- Embed this development into framework of morphological trees

# Thank you for your attention

?

## Main References

Aptoula, E., Lefèvre, S.: A comparative study on multivariate mathematical morphology. Pattern Recognition 40(11), 2914–2929 (November 2007)

Soille, P.: Constrained connectivity for hierarchical image partitioning and simplification. IEEE Trans. on Pattern Analysis and Machine Intelligence 30(7), 1132–1145 (July 2008)

Weber, J.: Segmentation morphologique interactive pour la fouille de séquences vidéo. Ph.D. thesis, Université de Strasbourg, France (2011)

Zanoguera, F.: Segmentation interactive d'images fixes et de séquences vidéo basée sur des hierarchies de partitions. Ph.D. thesis, Ecole des Mines de Paris (2001)